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First order Differential Eq.

$$y' = f(x, y)$$

1) seperable $\quad F(x, y) = F(x) \cdot G(y)$

2) $f(x, y) = F(ax + by + c) \Rightarrow$ let $Z = ax + by + c$

$\Rightarrow$ seperable $(x \,\&\, z)$

3) Homogeneous

$$f(x, y) = F\left(\frac{y}{x}\right)$$

let $v = \dfrac{y}{x} \Rightarrow y = xv \xrightarrow{\frac{d}{dx}} y' = v + xv'$

$\Rightarrow$ seperable $(x \,\&\, v)$

4) $f(x, y) = F\left(\dfrac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}\right)$

* Find the intersection point (h, k)

* let $x = t + h$ & $y = z + k \Rightarrow y' = \dfrac{dy}{dx} = \dfrac{dz}{dt}$

z'

$\Rightarrow$ Homogeneous $(t \,\&\, z)$

Note: If the lines are parallel Case ②

$\Rightarrow$ Case ②

$* F(ax+by+c)$ $\quad y' = \dfrac{2x-3y+1}{\sin(2x-3y+1)} \quad \ln(2x-3y+1)$

$* F(\frac{y}{x})$ $\quad xy' = \sqrt{x^2+y^2} \Longrightarrow y' = \sqrt{1+\frac{y}{x}}$

Examples p.15

1) $y' = \sqrt{4x+2y-1}$

let $z = 4x+2y-1 \xrightarrow[dx]{d} z' = 4 + 2y'$

D.E $\quad \dfrac{z'-4}{2} = \sqrt{z}$

$\Longrightarrow z' = (2\sqrt{z}+4) \Longrightarrow$ seperable

$\dfrac{dz}{dx} = 2\sqrt{z}+4 \Rightarrow \int \dfrac{dz}{2\sqrt{z}+4} = \int 1\, dx$

let $u^2 = z \Longrightarrow dz = 2u\, du$

$\int \dfrac{2u\, du}{2u+4} = \int \dfrac{u}{u+2}\, du = \int dx$

$= u - 2\ln(u+2) = \sqrt{z} - 2\ln(\sqrt{z}+2)$

Solution $\sqrt{z} - 2\ln(\sqrt{z}+2) = x + c$

$\left\{ \sqrt{4x+2y-1} - 2\ln(\sqrt{4x+2y-1}+2) = x + c \right\}$

homogeneous

13) $xy' - x e^{-\frac{y}{x}} = y \Longrightarrow y' = e^{-\frac{y}{x}} + \frac{y}{x}$

let $v = \frac{y}{x} \Longrightarrow y = xv \Longrightarrow y' = v + xv'$

D.E. $\cancel{v} + xv' = e^{-v} + \cancel{v} \Longrightarrow v' = \frac{1}{x} \cdot e^{-v}$
seperable

$\frac{dv}{dx} = \frac{1}{x} e^{-v} \Longrightarrow \int e^{v}\, dv = \int \frac{1}{x}\, dx$

$e^{v} = \ln x + \ln c \Longrightarrow e^{v} = \ln cx$

$\Longrightarrow e^{y/x} = \ln cx$

$\boxed{18}$ $(y'+1) \ln\left(\frac{y+x}{x+3}\right) = \frac{y+x}{x+3}$

* inter section $y+x=0$ & $x+3=0 \Longrightarrow (\underset{h}{-3}, \underset{k}{3})$

let $x = t-3$ & $y = z+3$

* D.E. $(z'+1) \ln\left(\frac{z+t}{t}\right) = \left(\frac{z+t}{t}\right)$

$(z'+1) \ln\left(\frac{z}{t}+1\right) = \frac{z}{t} + 1 \Longrightarrow$ Homog.

let $v = \frac{z}{t} \Longrightarrow vt = z \Longrightarrow z' = v + tv'$

D.E $(v+tv'+1)\ln(v+1)=v+1$

$$\frac{dv}{dt}=v'=\left(\frac{v+1}{\ln(v+1)}-(v+1)\right)\cdot\frac{1}{t}$$

$$\int\left(\frac{1}{\frac{1}{\ln(v+1)}-1}\right)\cdot\frac{1}{v+1}=\int\frac{1}{t}\,dt$$

$\Downarrow$

let $u=\ln(v+1)\Rightarrow\int\frac{1}{\frac{1}{u}-1}\,du$

$$=\int\frac{u}{1-u}\,du=-u-\ln(1-u)$$

$$-\ln(v+1)-\ln(1-\ln(v+1))=\ln ct$$

$$-\ln\left(\frac{z}{t}+1\right)-\ln\left(1-\ln\left(\frac{z}{t}+1\right)\right)=\ln ct$$

$$-\ln\left(\frac{y-3}{x+3}+1\right)-\ln\left(1-\ln\left(\frac{y-3}{x+3}+1\right)\right)=\ln(c(x+3))$$

$\boxed{17}$ $y' = 2\left(\frac{y+2}{x+y-1}\right)^2$

$y+2=0$

$x+y-1=0 \Rightarrow (3, -2)$ h K

let $x = t+3$ & $y = z-2$

* **D.E** $z' = 2\left(\frac{(z-2)+2}{(t+3)+(z-2)-1}\right)^2 = 2\left(\frac{z}{t+z}\right)^2$

$z' = 2\left(\frac{z/t}{1+z/t}\right)^2 \Rightarrow$ let $v = \frac{z}{t}$

$\Rightarrow vt = z \longrightarrow z' = v + tv'$

* **D.E** $v + t\underbrace{v'}_{\frac{dv}{dt}} = 2\left(\frac{v}{1+v}\right)^2 \Rightarrow v + tv' = \frac{2v^2}{1+2v+v^2}$

$t\left(\frac{dv}{dt}\right) = \frac{2v^2}{1+2v+v^2} - v = \frac{-v(1+v^2)}{1+2v+v^2}$

$\int \frac{1+2v+v^2}{-v(1+v^2)}\,dv = \int \frac{dt}{t} \Rightarrow \int \frac{1+v^2+2v}{-v(1+v^2)}\,dv = \int \frac{dt}{t}$

$-\int \frac{1}{v}\,dv - \int \frac{1}{1+v^2}\,dv = \ln(t) + c$

$-\ln(v) - 2\tan^{-1}(v) = \ln(t) + c$

$-\ln\left(\frac{z}{t}\right) - 2\tan^{-1}\left(\frac{z}{t}\right) = \ln(t) + c$

* $-\ln\left(\frac{y+2}{x-3}\right) - 2\tan^{-1}\left(\frac{y+2}{x-3}\right) = \ln(x-3) + C$ *

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