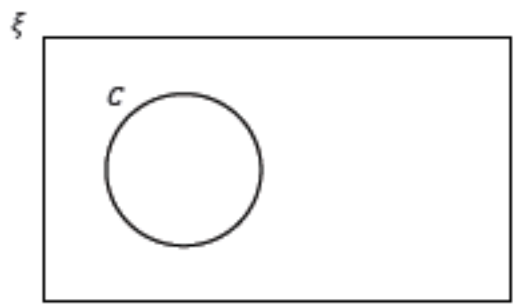
(a) The line $y = mx + c$ is tangent to the curve $x^2 + y^2 = 9$.

Show that $9m^2 = c^2 - 9$. [4]

(b) Find the range of values of k for which the function $y = 4x^2 - 2kx - k + 3$ is always positive for all real values of x. [4]

1.

(a) The universal set ξ and the sets A, B and C are given by



$\xi = x\!: x$ is an integer such that $3 \le x \le 42$,

$A = x\!: x$ is an even number,

$B = x\!: x$ is a prime number,

$C = x\!: x$ is a multiple of 7.

The Venn diagram shows the sets ξ and C.

(i) Copy the Venn diagram and add to your diagram the sets A and B. [2]

(ii) Write down the element x such that $x \in B \cap C$. [1]

(iii) Find $n(A' \cap B' \cap C)$. [1]

(b) Two sets P and Q are such that $n(\varepsilon) = 120, n(P) = 56$ and $n(Q) = 42$.

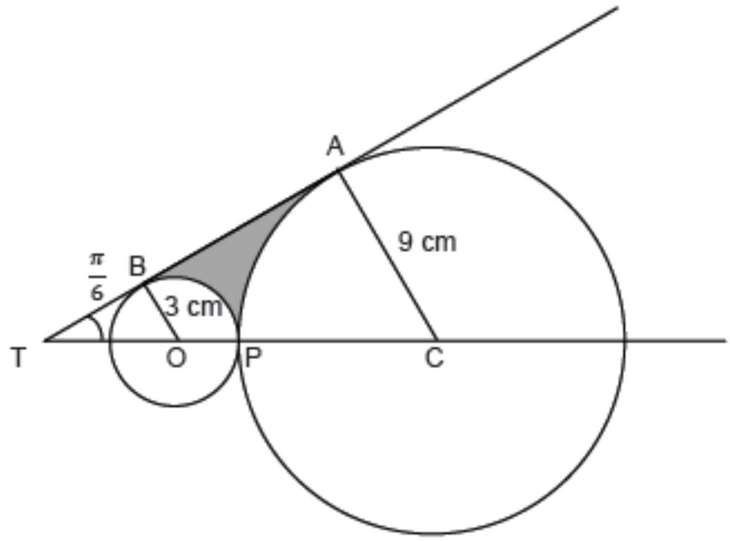
Find

(i) the greatest possible values of $n(P \cup Q)$, [1]

(ii) the smallest possible value of $n(P \cap Q')$. [1]

2.

The figure shows 2 circles with radii 3 cm and 9 cm and centre O and C respectively, touching externally at P. TBA is a tangent to the circles and $\angle ATC = \frac{\pi}{6}$ radians.



Calculate, correct to 2 decimal places,

(i) the length of AB, [2]

(ii) the length of the minor arc BP, [1]

(iii) the area of the shaded region. [3]

3.

The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	4.5	20.1	89.2	398.5	1800.3

It is known that x and y are related by the equation $Ay = e^{bx+1}$, where A and b are constants.

(i) Express this equation in a form suitable for drawing a straight line graph. [2]

(ii) Draw a straight line graph for the given data. [3]

(iii) Using the graph, estimate the value of A and of b. [2]

4.

The function f is defined for the domain $-2 \le x \le 2$, by $f(x) = 3 - 4x - 4x^2$.

(i) Express $f(x)$ in the form $a - b(x+c)^2$, where a, b and c are constants. [2]

(ii) State the coordinates of the turning point of this function. [1]

(iii) Find the range of f that corresponds to the given domain. [2]

(iv) Sketch the graph of $y = f(x)$ for this domain of x, indicating clearly all the turning points and y-intercept. [4]

(v) Another function $g(x) = 3 - 4x - 4x^2$ is defined for the domain $x \ge k$. State the minimum value of k, for which g has an inverse. [1]

Answer

(a) $y = mx + c$ ----- 1

$x^2 + y^2 = 9$ ----- 2

Sub 1 into 2,

$x^2 + (mx + c)^2 = 9$

$x^2 + m^2x^2 + 2mcx + c^2 = 9$

$(m^2 + 1)x^2 + (2mc)x + (c^2 - 9) = 0$

Since the line $y = mx + c$ is tangent to the curve $x^2 + y^2 = 9$,

$D, (2mc)^2 - 4(m^2 + 1)(c^2 - 9) = 0$

$4m^2c^2 - 4(m^2c^2 - 9m^2 + c^2 - 9) = 0$

$36m^2 - 4c^2 + 36 = 0$

$9m^2 = c^2 - 9$ (shown)

(b) Since the function is always positive,

$D, (-2k)^2 - 4(4)(3 - k) < 0$

$4k^2 - 48 + 16k < 0$

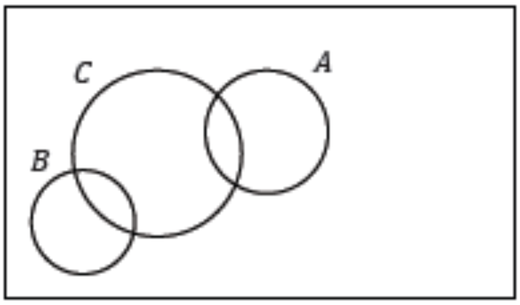
$k^2 + 4k - 12 < 0$

$(k + 6)(k - 2) < 0$

$-6 < k < 2$

1.

(a) (i) ξ



(ii) $x:7$

(iii) $n = 2$

2.

(b) (i) greatest possible value $= 56 + 42 = 98$

(ii) smallest possible values $= 56 - 42 = 14$

(i) $AT = \frac{9}{\tan\frac{\pi}{6}}$

$BT = \frac{3}{\tan\frac{\pi}{6}}$

$AB = AT - BT$

$= 10.4$ cm (correct to 3 sig. fig.)

(ii) $\angle BOP = \pi - (\pi - \frac{\pi}{2} - \frac{\pi}{6})$

$= \frac{2\pi}{3}$

Length of minor arc $BP = \frac{2\pi}{3} \div 2\pi \times \pi(6)$

$= 6.28$ cm (correct to 3 sig. fig.)

(iii) Area of $\Delta ATC = \frac{1}{2} \times \frac{9}{\tan\frac{\pi}{6}} \times 9$

Area of shaded region

$= \frac{1}{2} \times \frac{9}{\tan\frac{\pi}{6}} \times 9 - \frac{1}{2} \times \frac{3}{\tan\frac{\pi}{6}} \times 3 - \frac{2\pi}{3} \div 2\pi \times \pi(3)^2 - \frac{\pi}{3} \div 2\pi \times \pi(9)^2$

$= 10.5$ cm^2 (correct to 3 sig. fig.)

3.

(i) $Ay = e^{bx+1}$

$\ln Ay = bx + 1$

$\ln y = bx + 1 - \ln A$

(ii) Plot $\ln y$ against x.

(iii) $A = e(2.72)$

$b = 1.5$

4.

(i) $f(x) = 3 - 4(x^2 + x)$

$= 3 - 4\left[\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$

$= 4 - 4\left(x + \frac{1}{2}\right)^2$

5.

(ii) Turning point is $\left(-\frac{1}{2}, 4\right)$

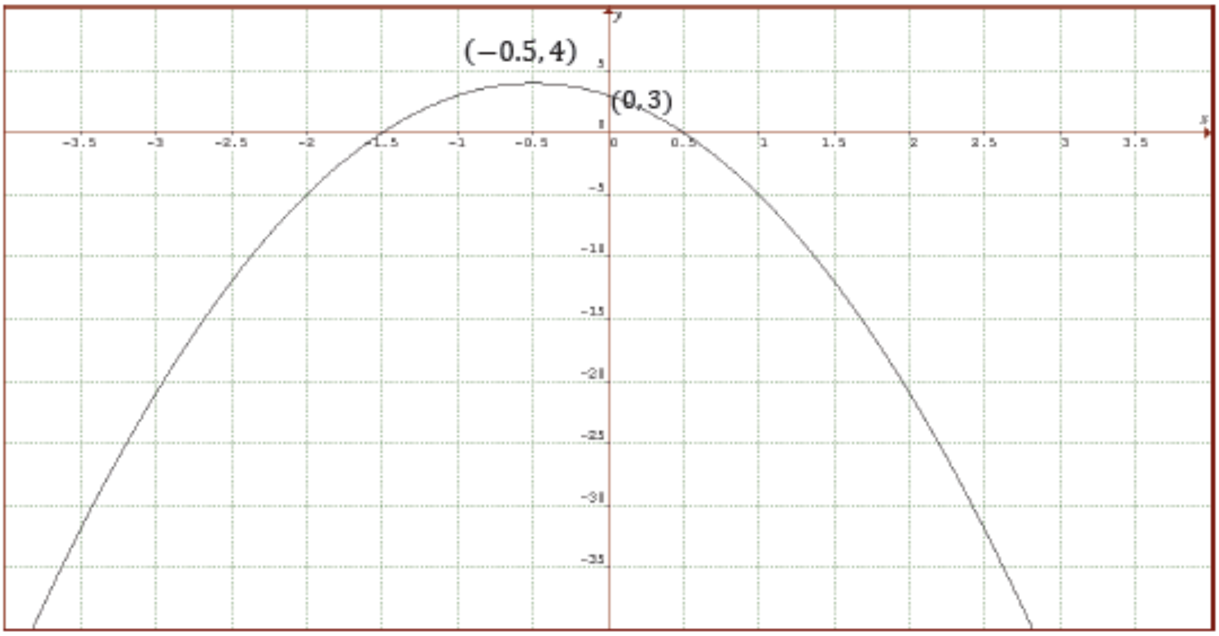
(iii) when $x = 2$,

$f(x) = 3 - 4(2) - 4(2)^2$

$= -21$

$-21 \leq f(x) \leq 4$

(iv)



(v) $k = -\frac{1}{2}$