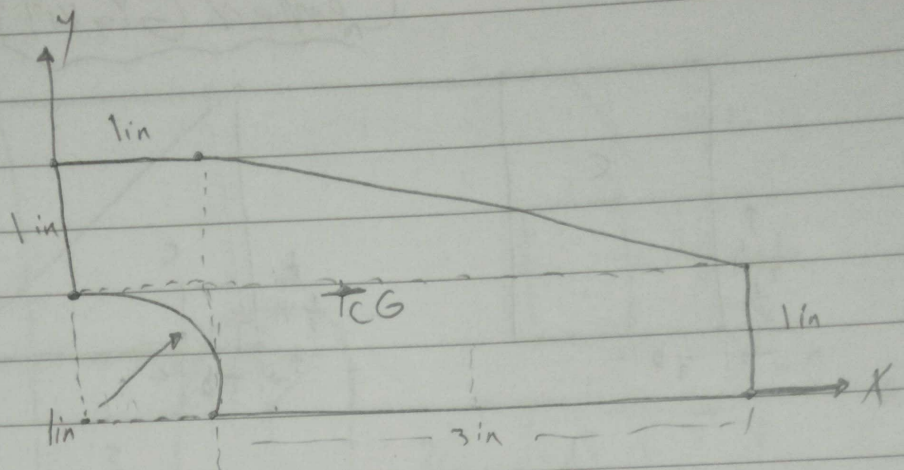
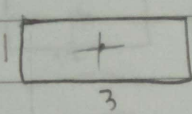
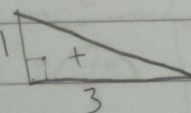
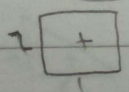



Date

/ /



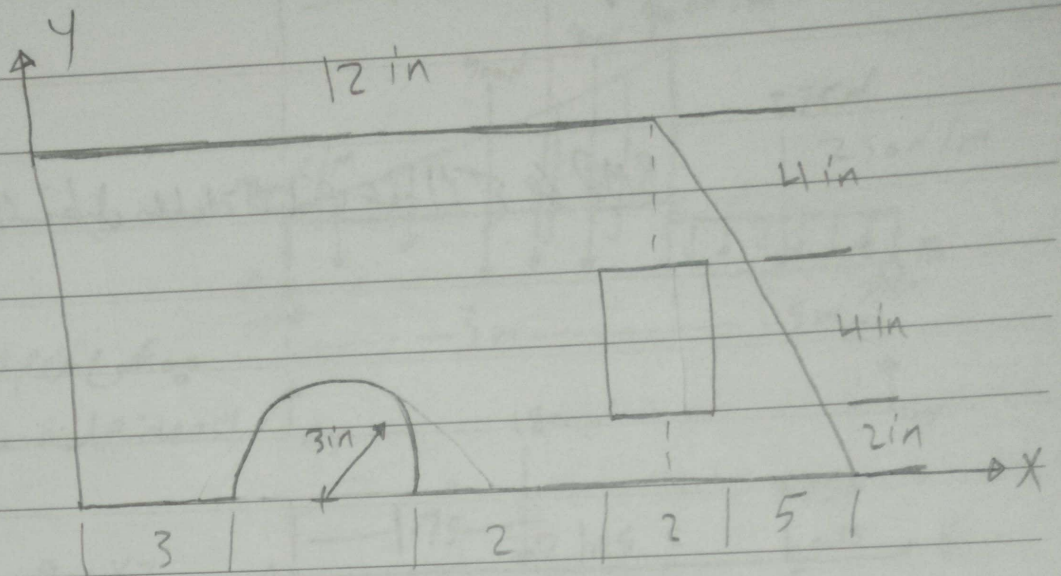
Parts	A_i	X_i	Y_i	AX_i	AY_i
	$3 \times 1 = 3$	2.5	0.5	7.5	$\frac{3}{2}$
	$\frac{1}{2} \times 1 \times 3 = \frac{3}{2}$	2	$\frac{4}{3}$	3	2
	$1 \times 1 = 1$	0.5	1	1	2
	$-\pi \times 1^2 \times \frac{1}{4} = -\frac{\pi}{4}$	$\frac{41}{3\pi}$	$\frac{3}{3\pi}$	$-\frac{1}{3}$	$-\frac{1}{4}$
Σ	5.7			11.2	5.25

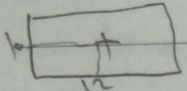
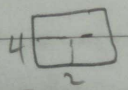
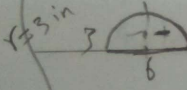
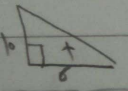
$$\bar{X} = \frac{\Sigma X_i A_i}{\Sigma A_i} = 1.96 \text{ in}$$

$$\bar{Y} = \frac{\Sigma Y_i A_i}{\Sigma A_i} = 0.92 \text{ in}$$

Date

1 / 1



PARTS	A_i	x_i	y_i	Ax_i	Ay_i
	120	6	5	720	600
	-8	12	4	-96	-32
	$-\frac{9}{2}\pi$	6	$\frac{4}{\pi}$	-27	-18
	30	14	$\frac{10}{3}$	420	100
Σ	127.9			959.2	650

$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{959.2}{127.9} = 7.5 \text{ in}$$

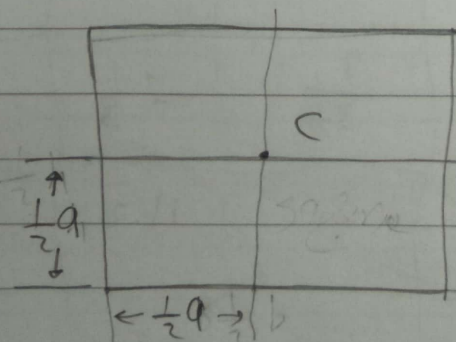
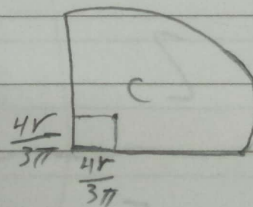
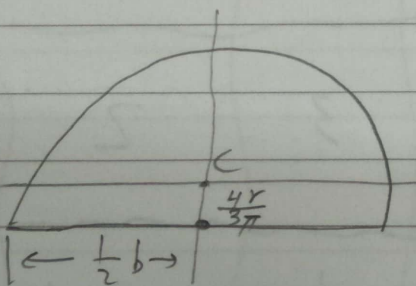
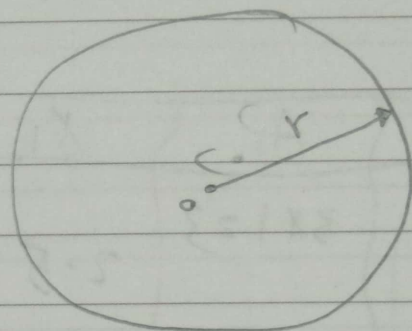
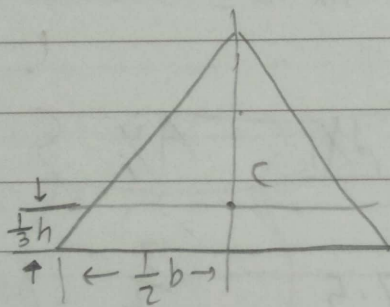
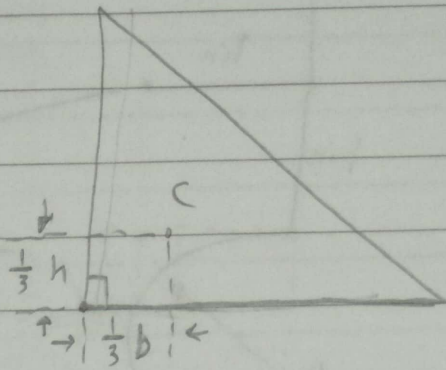
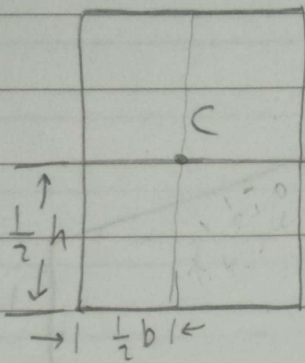
$$\bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{650}{127.9} = 5.08 \text{ in}$$



Date

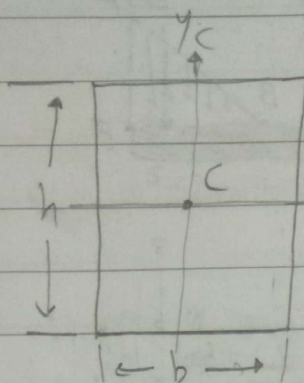
1 1

Center Gravity



Date

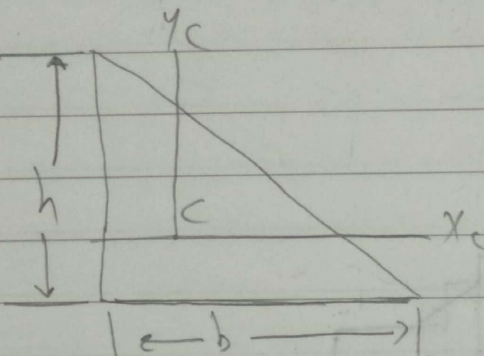
Area moment of inertia



$$I_{xc} = \frac{b \times h^3}{12}$$

$$I_{yc} = \frac{h \times b^3}{12}$$

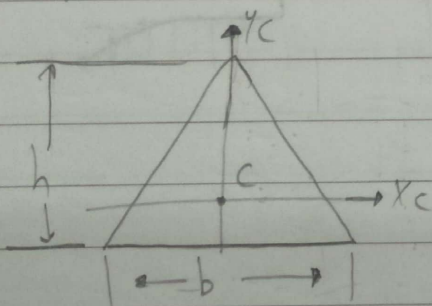
موازي المحاور
15



$$I_{xc} = \frac{b \times h^3}{36}$$

$$I_{yc} = \frac{h \times b^3}{36}$$

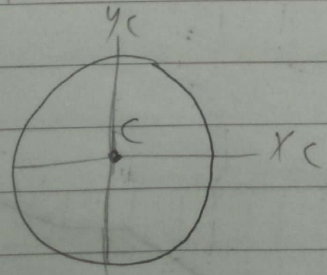
موازي المحاور
37



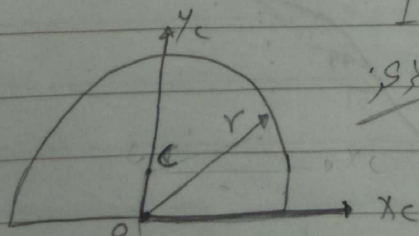
$$I_{xc} = \frac{b \times h^3}{36}$$

$$I_{yc} = \frac{h \times b^3}{48}$$

I_{xc} : ممان القصور حول المحور x
عند نقطة أفقية من مركز الثقل
محيط مركز نقطة C والثقل
تلك القوة من أعلى وتسمى (C)
أيضاً المعرف قوة تادلة عند
منه النقطة (C)

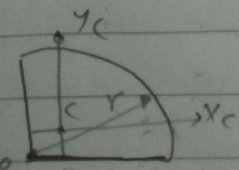


$$I_{xc} = I_{yc} = \frac{\pi r^4}{4}$$

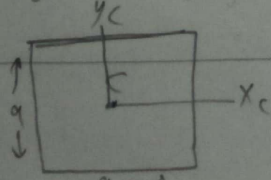


$$I_{xc} = I_{yc} = \frac{\pi r^4}{8}$$

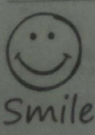
عند المركز



$$I_{xc} = I_{yc} = \frac{\pi r^4}{16}$$

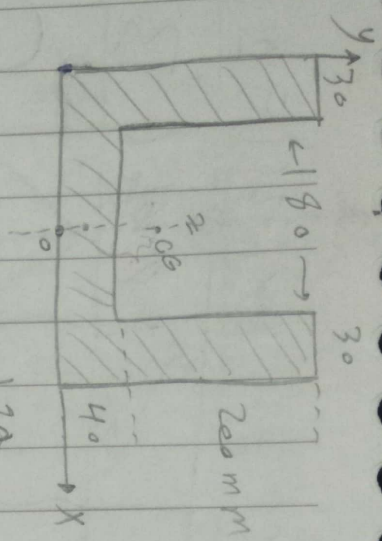


$$I_{xc} = I_{yc} = \frac{a^4}{12}$$



Smile

Nawar



المساحة الكلية = 200 × 180 + 240 × 120 = 36000 + 28800 = 64800 mm²
 المساحة الكلية = 64800 mm²
 المساحة الكلية = 64800 mm²
 المساحة الكلية = 64800 mm²

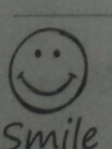
Parts	A _i	X _i	y _i	A _i · X _i	A _i · y _i	I _{xci}	A · d _i ²	I _{yci}	A · d _i ²
+	240 × 120 = 57600	120	120	240 × 120 = 6912000	240 × 120 = 6912000	$\frac{240^3}{12} = 2764800$	$57600 \times 53.3^2 = 1642562.7$	$\frac{240^3}{12} = 2764800$	$57600 \times 0 = 0$
-	200 × 180 = 36000	120	40	-200 × 180 = -4320000	-200 × 180 = -5040000	$\frac{200^3}{12} = 666666.7$	$-36000 \times 53.3^2 = -102272040$	$\frac{200^3}{12} = 666666.7$	$-36000 \times 0 = 0$
Σ	21600			2592000	1872000	15648000	-38399976	17928000	0

$$\bar{X} = \frac{\sum A_i \cdot X_i}{\sum A_i} = \frac{2592000}{21600} = 120 \text{ mm}$$

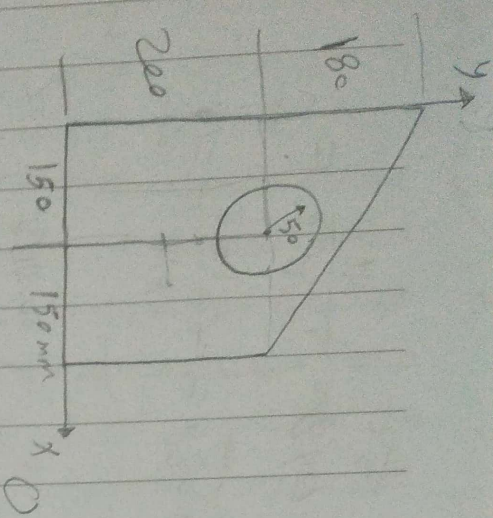
$$\bar{Y} = \frac{\sum A_i \cdot y_i}{\sum A_i} = \frac{1872000}{21600} = 86.7 \text{ mm}$$

$$I_{xc} = 11808000 \text{ mm}^4$$

$$I_{yc} = 17928000 \text{ mm}^4$$



Date _____



Part 5

A

—

91

2008300
-10000

50

1

$$= 9 \times 10^6 = 6 \times 10^6$$
$$\begin{array}{r} 12 \\ 2 \times 10 \\ 8 \end{array}$$
$$= 6008$$
$$= 45 \times 10$$
$$= 135 \times 10$$

$$= 27000$$

00

266

$$\begin{array}{r} X|oc5 \\ 27X|o5 \end{array}$$
$$x^2bc = 762 \times 10^4$$
$$36 \times 10 = 360$$
$$= 1000000$$
$$= 135 \times 10$$
$$- \pi \cdot 50^2$$

150


22

5-1178097

$\lambda = -1570796$

$$= -156250$$

-1562500



50


3

194.66 mm

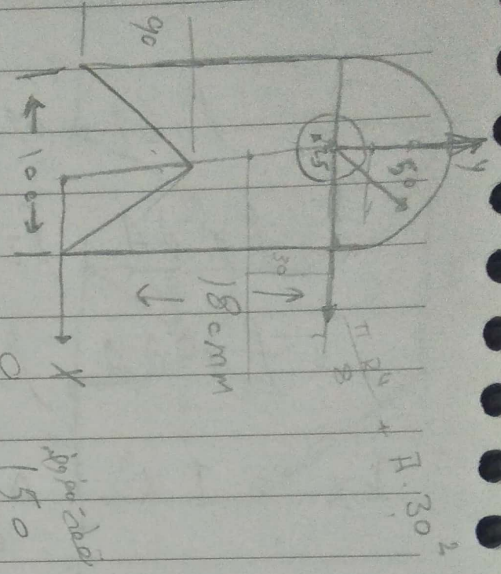
$$X = 132.9 \text{ mm}$$

IXC

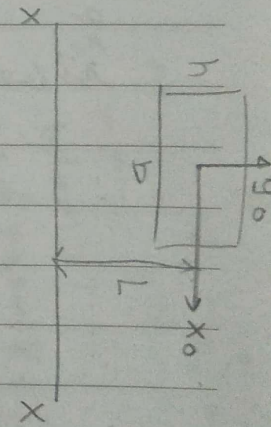
79c



Date

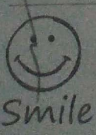


Part	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$	I_{xc_i}	$A \cdot d_{x_i}^2$	I_{yc_i}	$A \cdot d_{y_i}^2$
1	$100 \times 18 = 1800$	0	9	0	$1800 \times 9 = 162000$	$\frac{100 \times 18^3}{12} = 486 \times 10^3$	$1800 \times 66^2 = 648 \times 10^5$	$\frac{18 \times 100^3}{12} = 15 \times 10^6$	$1800 \times 0 = 0$
2	$\frac{1}{2} \times \pi \times 15^2 = 1250\pi$	0	$18 + \frac{4 \times 10}{3} = 26.67$	0	$1250\pi \times 26.67 = 251250\pi$	$\frac{\pi \times 15^4}{8} = 2454369$	$1250\pi \times 30^2 = 3534291$	$\frac{\pi \times 15^4}{8} = 2454369$	$1250\pi \times 0 = 0$
3	$\frac{1}{2} \times 100 \times 10 = 4500$	0	3	0	$4500 \times 3 = 13500$	$\frac{100 \times 10^3}{12} = 2625000$	$4500 \times 126^2 = 648 \times 10^5$	$\frac{90 \times 100^3}{12} = 1875000$	$4500 \times 0 = 0$
4	$\frac{1}{2} \times \pi \times 10^2 = 25\pi$	0	19	0	$25\pi \times 180 = 4500\pi$	$\frac{\pi \times 10^4}{8} = 39270$	$25\pi \times 30^2 = 23562$	$\frac{\pi \times 10^4}{8} = 39270$	$25\pi \times 0 = 0$
5	15463	0	132	0	$15463 \times 132 = 2042396$	548976	116745	15272573	15272573
Total	15463	0	132	0	2042396	548976	116745	15272573	15272573



$$I_x = \frac{b \cdot h^3}{12} + b \cdot h \cdot d^2$$

Nawar



Date / /

$$I_x = \int \bar{y}^2 dA$$

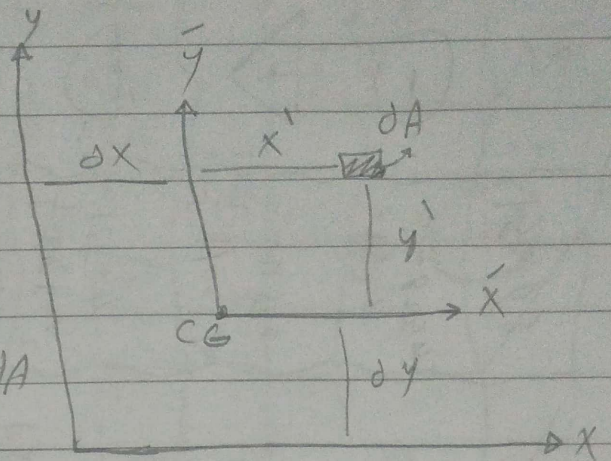
$$I_x = \int (y' + \delta y)^2 dA$$

$$= \int y'^2 dA + \int 2y' \delta y dA + \int \delta y^2 dA$$

$$= \int y'^2 dA + \int 2y' \delta y dA + \int \delta y^2 dA$$

$$I_x = I_{x'} + A \delta y^2$$

$$I_y = I_{y'} + A \delta x^2$$

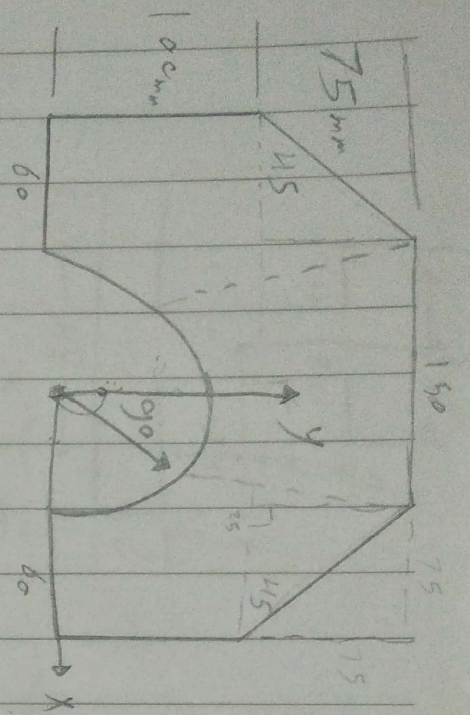


$$\begin{aligned} \delta I_x &= \bar{y}^2 \delta A \\ \delta I_y &= \bar{x}^2 \delta A \end{aligned}$$

$$\begin{aligned} I_x &= \int \bar{y}^2 dA \\ I_y &= \int \bar{x}^2 dA \end{aligned}$$

نتج من هذه الدقة عند التقريب وتكون النتيجة هي أن المساحة المربعة أو المثلثية أو غيرها
تكون عند مركزها الناتج النهائي.

Date / /



$$\bar{X} = 0$$

$$\bar{Y} = 95.6 \text{ mm}$$

$$KZ = \sqrt{K_x^2 + K_y^2}$$

$$K = \sqrt{\frac{I}{A}}$$

$$K_x = \sqrt{\frac{I_x}{A}}$$

$$K_y = \sqrt{\frac{I_y}{A}}$$

Part 1

A_1

X_1

Y_1

$A_1 \cdot X_1$

$A_1 \cdot Y_1$

I_{xc1}

$A d_{y1}^2$

I_{yc1}

$A d_{x1}^2$

$$300 \times 100 = 30000$$

$$0$$

$$50$$

$$0$$

$$1500000$$

$$\frac{300 \times 100^3}{12} = 25 \times 10^6$$

$$30000 \times 50^2 = 75 \times 10^6$$

$$\frac{100 \times 300^3}{12} = 225 \times 10^6$$

$$30000 \times 300^2 = 2700 \times 10^6$$

Part 2

$$\frac{1}{2} \pi r^2 = 4050 \pi$$

$$0$$

$$\frac{4 \times 90}{3 \pi} = 38$$

$$0$$

$$-153900 \pi$$

$$\frac{\pi r^4}{8} = -25764986$$

$$-4050 \pi \times 0 = 0$$

$$\frac{\pi r^4}{8} = -25764986$$

$$-4050 \pi \times 0 = 0$$

Part 3

$$150 \times 75 = 11250$$

$$0$$

$$137.5$$

$$0$$

$$1546875$$

$$\frac{150 \times 75^3}{12} = 5273437$$

$$11250 \times 137.5^2 = 212695312$$

$$\frac{75 \times 150^3}{12} = 21093750$$

$$11250 \times 0 = 0$$

$$\frac{1}{2} \times 75 \times 75 = 2812.5$$

$$-100$$

$$125$$

$$-281200$$

$$351500$$

$$\frac{75^4}{36} = 878906$$

$$2812.5 \times 125^2 = 43945312.5$$

$$\frac{75^4}{36} = 878906$$

$$2812.5 \times 100^2 = 28125000$$

Part 4

$$\frac{1}{2} \times 75 \times 75 = 2812.5$$

$$100$$

$$125$$

$$281200$$

$$351500$$

$$\frac{75^4}{36} = 878906$$

$$2812.5 \times 125^2 = 43945312.5$$

$$\frac{75^4}{36} = 878906$$

$$2812.5 \times 100^2 = 28125000$$

$$34150$$

$$0$$

$$125$$

$$281200$$

$$32663875$$

$$6266263$$

$$375585931$$

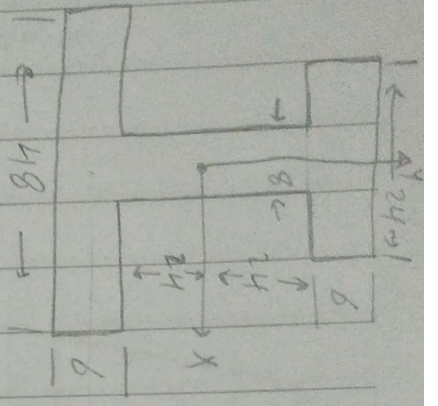
$$222086576$$

$$56250000$$

Smile

Nawar

Date 1 / 1



Part	A	I_{X1}	Adj.	I_{Y1}	Adj.
1	48×8 384	$\frac{8 \times 48^3}{12}$ 75728	384×0 0	$\frac{48 \times 8^3}{12}$ 2048	384×0 0
2	24×6 144	$\frac{24 \times 6^3}{12}$ 432	144×27^2 104976	$\frac{6 \times 24^3}{12}$ 6912	144×0 0
3	6×48 288	$\frac{48 \times 6^3}{12}$ 864	288×27^2 209952	$\frac{6 \times 48^3}{12}$ 55296	288×0 0
Total		75024	314928	64256	0
Sum			389952 mm ⁴		64256 mm ⁴

$$K_x = \sqrt{\frac{I_x}{A}}$$

$$K_y = \sqrt{\frac{I_y}{A}}$$